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Phys. Scr. **77** (2008) 065002 (6pp) doi:10.1088/0031-8949/77/06/065002

Non-Hermitian interaction of matter and light

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Received 26 May 2007
Accepted for publication 10 April 2008
Published 21 May 2008
Online at stacks.iop.org/PhysScr/77/065002

Abstract
We investigate the non-Hermitian Hamiltonian which governs the system including two-level atom and electromagnetical field with a circular polarization vector. We find the Hamiltonian by using dipole approximation and rotating wave approximation (RWA), which lead to a non-Hermitian Hamiltonian. We solve the time-independent non-Hermitian Hamiltonian and obtain real eigenvalues of energy for this system. Finally, by solving the Schrödinger equation in the pseudo interaction picture, we show that our results and the results of Jaynes–Cummings (JC) Hamiltonian are in excellent agreement.

PACS numbers: 42.50.−P, 32.30.Jc

(Some figures in this article are in colour only in the electronic version.)

1. Introduction

Quantum optics provides the ideal area to deal with the interaction of radiation and matter. It is correct to say that the foundation of quantum optics and particularly the interaction between radiation and matter are constructed on the concept of a few-level atom. Indeed, the most important and well-known concept which has been introduced so far in this category is the two-level atom. A large number of physical concepts can be studied by such a model, the so-called Jaynes–Cummings (JC) model, describing a two-level atom interacting with a single mode electromagnetic (EM) field [1–3]. This simplicity allows exact analytic application of the fundamental laws of quantum mechanics and electrodynamics. Moreover, its exact solvability in the rotating wave approximation (RWA) exhibits interesting quantum mechanical effects like the collapses and revivals of Rabi oscillations [1]. It is well known that this model is based on some relevant approximations which are well verified in experiments. In fact the JC model, which is obtained versus one of the standard axioms of quantum mechanics, is to be considered as self-adjoint operators, so that the corresponding eigenvalues are real and the time evolution of the eigenstates is unitary, so that to satisfy the hermiticity of the Hamiltonian for the case of complex polarization vector, the $\hat{\epsilon}$ and its conjugation $\hat{\epsilon}^\star$ are both used in the interaction term of the Hamiltonian.

On the other hand, a new viewpoint emerging in the current literature is that although the condition of hermiticity is sufficient to have a unitary theory with real eigenvalues, it is not necessary. This was mainly initiated by Bender and Boettcher's researches that with properly defined boundary conditions the eigenvalues of the Hamiltonian $H = p^2 - x^2(\mathrm{i}x)^n$ $(n \geqslant 0)$ are real, discrete and positive. However the reality of the eigenvalues is a consequence of unbroken $\mathcal{PT}$ symmetry, $[H, \mathcal{PT}] = 0$ ($\mathcal{P}$ is the parity, and $\mathcal{T}$ is the time reversal operator) [4–6]. The spectrum appear in complex-conjugate pairs, if the $\mathcal{PT}$ symmetry is broken spontaneously [7–17]. In another approach [18–20], it has been shown that the reality of eigenvalues of non-Hermitian Hamiltonian is due to the so-called pseudo-hermiticity properties of the Hamiltonian, which is defined by $\eta H \eta^{-1} = H^\dagger$, where η is a linear, Hermitian and invertible operator. If η is to be an antilinear, Hermitian and invertible operator and $\eta H \eta^{-1} = H^\dagger$, the Hamiltonian is called anti-pseudo Hermitian. Therefore, this shows that in some physical models the Hermitian is not required, i.e. there is no need to limit some of physical models according to Hermitian interaction. The authors of [21, 22] have studied the pseudo supersymmetry and quadratic pseudo supersymmetry in two-level systems, also.

In this work, we obtain the Hamiltonian which was considered in [21]. In fact we show that the interaction of EM field with circular polarization by an atom can create that non-Hermitian Hamiltonian which was considered in [21]. It is necessary to mention that in ordinary formalism of quantum mechanics to fulfil the hermiticity of the Hamiltonian for circular polarization, $\hat{\epsilon}$ and $\hat{\epsilon}^{\star}$ are used simultaneously in the interaction term of the Hamiltonian; i.e. using $\hat{\epsilon}$ and $\hat{\epsilon}^{\star}$ simultaneously is only for making the Hamiltonian Hermitian. Now there is a question. What must be done if we want the Hamiltonian to be non-Hermitian? We believe that, here, as in the linear polarization case, we can use ϵ only. The result is a non-Hermitian Hamiltonian with some extra symmetries. To study the model physically, we need to compare eigenstate, eigenvalues and the evolution of this system with the one that was obtained from ordinary quantum mechanics or experimental data. The results of this paper show that this model is a description for many problems in physics and optics. That makes this study important.

It is notable that the resulting Hamiltonian by this method is not Hermitian, and it does not have the $\mathcal{PT}$ symmetry but it has another symmetry which is expressed in this way $\mathcal{P}\sigma_z$ (where $\mathcal{P}$ is the parity and σ_z is the Pauli matrix) in other words

$$[\mathcal{H}, \mathcal{P}\sigma_z] = 0.$$

Also, we solve the time-independent state of this model and we obtain that the eigenvalues of this Hamiltonian are real. We show that the Hamiltonian of this model is σ_z-pseudo-Hermitian Hamiltonian, and then the eigenstates of them are orthonormal with respect to σ_z-pseudo inner product. So, this shows that there is another Hamiltonian (rather than JC Hamiltonian) which well describes the interaction of two-level atoms with EM waves. It is remarkable that the difference between these two Hamiltonians is not in the shape of interaction, but in being Hermitian or not, and this is of great importance because we can describe the weak points of Hermitian models by this Hamiltonian. Finally, by solving the Schrödinger equation in the pseudo interaction picture, we show that our results and the results of JC Hamiltonian are in excellent agreement.

The scheme of this paper is as follows: in section 2, using the dipole approximation and then the RWA approximation, we obtain the non-Hermitian Hamiltonian that describes the interaction between the EM field and an atom. In section 3, we show that this Hamiltonian is a non-Hermitian operator for an EM field with circular polarization which is not $\mathcal{PT}$ invariant but σ_z-pseudo-Hermitian. In section 4, we consider the stationary state of the model and obtain the energy spectrum and eigenstates of it. In section 5, we solve the evolution of the model and obtain the physical quantity of that, and finally in section 6 we write the conclusion of the paper.

2. Atom–field interaction

The Hamiltonian that describes such a system with an atom interacting with an EM field is

$$H = H_{\mathrm{a}} + H_{\mathrm{f}} + H_{\mathrm{int}}, \tag{1}$$

where H_{a} is the atom's Hamiltonian in the presence of an EM field, H_{f} is the Hamiltonian of the free EM field and H_{int} describes the interaction of the atom with light. One can explain the free EM field Hamiltonian in terms of photon annihilation and creation operator as

$$H_f = \sum_k \hbar\nu_k \left(a_k^{\dagger} a_k + \frac{1}{2}\right), \tag{2}$$

where ν_k is the frequency of the kth EM mode, and $a_k(a_k^{\dagger})$ is the photon annihilation (creation) operator with the frequency ν_k. For H_{a} we have

$$H_{\mathrm{a}}|j\rangle = E_j|j\rangle, \tag{3}$$

where $|j\rangle$ is the eigenket of H_{a} of the free atom, where $|j\rangle$s make the complete set that describe the internal state of the atom,

$$\sum_j |j\rangle\langle j| = \mathbf{1}. \tag{4}$$

Therefore, we define the transition operator to the jth state as

$$\sigma_{jl} = |j\rangle\langle l|, \tag{5}$$

so

$$H_{\mathrm{a}} = \sum_j E_j \sigma_{jj}. \tag{6}$$

By rewriting the interaction Hamiltonian (H_{int}) on the basis of H_{a}, one can obtain

$$H_{\mathrm{int}} = \left(\sum_{jl} -\tilde{P}_{jl}\sigma_{jl}\right)\tilde{E}, \tag{7}$$

where $\tilde{P}_{jl} = \langle j| - e\tilde{r}|l\rangle$ is the matrix element of the dipole moment and $\tilde{r}$ is the position vector. $j = l$, $\tilde{P}_{jl} = \tilde{o}\ \tilde{P}_{jl} = \tilde{P}_{lj}$. Obviously, by using the second quantization approach, one can rewrite the electrical field in the atom position as

$$\tilde{E} = \sum_k \hat{\epsilon}_k \mathcal{E}_k (a_k + a_k^{\dagger}), \tag{8}$$

where $\mathcal{E}_k = (\frac{\hbar\nu_k}{2\varepsilon_0 V})^{1/2}$ and $\hat{\epsilon}_k$ is the polarization vector of the kth mode. So, we obtain the total Hamiltonian as

$$H_{\mathrm{tot}} = \sum_j E_j\sigma_{jj} + \sum_k \hbar\nu_k \left(a_k^{\dagger}a_k + \frac{1}{2}\right) + \hbar\sum_{jl}\sum_k \hat{g}_k^{jl}\sigma_{jl}(a_k + a_k^{\dagger}), \tag{9}$$

in which

$$\hat{g}_k^{jl} = -\frac{\mathcal{E}_k\hat{\epsilon}_k\cdot\tilde{P}_{jl}}{\hbar} \tag{10}$$

is the complex coupling parameter associated with the coupling of the field mode to the atomic transition. Equation (9) shows the interaction between the N-level atom and the k mode EM field. Let us consider a two-level atom interacting with a single mode EM field. We assume $|g\rangle$, $|e\rangle$ are eigenstates of the two-level atom, and the difference between energy of these states is $\hbar\omega$ and the EM field energy is $\hbar\nu$. Consequently, equation (9) gives

$$\begin{aligned} H = {} & E_g\sigma_{gg} + E_e\sigma_{ee} + \hbar\nu(a^{\dagger}a + \tfrac{1}{2}) \\ & + \hbar(\hat{g}^{ge}\sigma_{ge} + \hat{g}^{eg}\sigma_{eg})(a + a^{\dagger}). \end{aligned} \tag{11}$$

Apart from some unimportant constants, one can rewrite equation (11) as

$$H=\frac{\hbar\omega\sigma_z}{2}+\hbar\nu(a^\dagger a)+\hbar(\hat{g}^{ge}\sigma^++\hat{g}^{eg}\sigma^-)(a+a^\dagger), \qquad (12)$$

where $\sigma_z:=\sigma_{gg}-\sigma_{ee}$, $\sigma^+:=\sigma_{ge}$, $\sigma^-:=\sigma_{eg}$. From equation (11) the interaction Hamiltonian is

$$\begin{aligned}H_{\rm int}&=\hbar(\hat{g}^{ge}\sigma^++\hat{g}^{eg}\sigma^-)(a+a^\dagger)\\&=\hbar\left\{\hat{g}^{ge}\sigma^+a+\hat{g}^{ge}\sigma^+a^\dagger+\hat{g}^{eg}\sigma^-a+\hat{g}^{eg}\sigma^-a^\dagger\right\},\end{aligned} \qquad (13)$$

this interaction is an adiabatic interaction, so the energy should be conserved. The second term shows that one photon creates and the atom goes to a higher level, the third term shows that one photon annihilates and the atom goes to a lower level. We can remove these terms regarding energy conservation. So we have

$$H_{\rm int}=\hbar\left\{\hat{g}^{ge}\sigma^+a+\hat{g}^{eg}\sigma^-a^\dagger\right\}. \qquad (14)$$

It is clearly seen that we can write the interaction Hamiltonian in the interaction picture as

$$H_{\rm int}=\hbar(\hat{g}^{ge}\,{\rm e}^{-{\rm i}\Delta t}\sigma^+a+\hat{g}^{eg}\,{\rm e}^{{\rm i}\Delta t}\sigma^-a^\dagger), \qquad (15)$$

where $\Delta=\omega-\nu$ is the detuning parameter. Here $\hat{g}^{ge}(\hat{g}^{eg})$ is the complex coupling constant of the coupling of the field and atomic transition $|g\rangle\longrightarrow|e\rangle(|e\rangle\longrightarrow|g\rangle)$ [23–25].

3. Interaction between an atom and an EM field with circular polarization in the non-Hermitian Hamiltonian model

Now, let us consider an EM field with circular polarization vector as

$$\hat{\epsilon}=\frac{1}{\sqrt{2}}(\tilde{e}_x\pm{\rm i}\tilde{e}_y). \qquad (16)$$

In this case, the quantities $\hat{g}_{ge}$ and $\hat{g}_{eg}$ are

$$\hat{g}_{eg}=\frac{\mathcal{E}}{\hbar\sqrt{2}}(\tilde{e}_x\cdot\tilde{P}\pm{\rm i}\tilde{e}_y\cdot\tilde{P}) \qquad (17)$$

and

$$\hat{g}_{eg}=\frac{\mathcal{E}}{\hbar\sqrt{2}}(\tilde{e}_x\cdot\tilde{P}\pm{\rm i}\tilde{e}_y\cdot\tilde{P}), \qquad (18)$$

respectively, where $\tilde{P}=\langle g|e\tilde{r}|e\rangle=-\tilde{P}_{ge}=-\tilde{P}_{eg}$. Using the expressions (15) and (16) and substituting them in the interaction Hamiltonian (13), we have

$$\begin{aligned}H_{\rm int}=&\hbar g_x(\sigma^-a^\dagger\,{\rm e}^{{\rm i}\Delta t}+\sigma^+a\,{\rm e}^{-{\rm i}\Delta t})\\&+{\rm i}\hbar g_y(\sigma^+a\,{\rm e}^{-{\rm i}\Delta t}+\sigma^-a^\dagger\,{\rm e}^{{\rm i}\Delta t}),\end{aligned} \qquad (19)$$

where $g_x=\frac{\mathcal{E}}{\hbar\sqrt{2}}p_x$, $g_y=\frac{\mathcal{E}}{\hbar\sqrt{2}}p_y$, where $p_i=\tilde{e}_i.\tilde{P}$ $(i=x,y)$. It is remarkable that, to satisfy the hermiticity of the Hamiltonian, for the case of complex polarization vector (e.g. circular polarization) the $\hat{\epsilon}$ and its conjugation $\hat{\epsilon}^\star$ are both used in the interaction term of the Hamiltonian. But we do not want the interaction Hamiltonian to be Hermitian, so that the hermiticity of the Hamiltonian and consequently, conjugation of polarization vector $\hat{\epsilon}^\star$ is not necessary. Therefore, the interaction Hamiltonian which we have obtained from these assumptions is not a Hermitian operator, i.e.

$$\begin{aligned}H_{\rm int}^\dagger=&\hbar g_x(\sigma^+a{\rm e}^{-{\rm i}\Delta t}+\sigma^-a^\dagger{\rm e}^{{\rm i}\Delta t})-{\rm i}\hbar g_y(\sigma^+a{\rm e}^{-{\rm i}\Delta t}+\sigma^-a^\dagger{\rm e}^{{\rm i}\Delta t})\\&\neq H_{\rm int}.\end{aligned} \qquad (20)$$

For $g_y=0$, which is equivalent to a linear polarization vector in the $\hat{x}$-direction, the interaction Hamiltonian is

$$H_{\rm int}=\hbar g_x(\sigma^-a^\dagger{\rm e}^{-{\rm i}\Delta t}+\sigma^+a{\rm e}^{-{\rm i}\Delta t}), \qquad (21)$$

where (21) is a Hermitian operator and, in this case, the total Hamiltonian (12) is Hermitian and is called the JC Hamiltonian. For the case $g_y=0$ the total Hamiltonian takes the JC form, and the JC Hamiltonian has already been solved completely. We consider the total Hamiltonian with the non-Hermitian part of interaction only, i.e. $(g_x=0)$

$$H_{\rm tot}=\frac{\hbar\nu}{2}\sigma_z+a^\dagger a\hbar\nu+H_{\rm int}, \qquad (22)$$

$$H_{\rm int}={\rm i}\hbar g_y(\sigma^+a{\rm e}^{-{\rm i}\Delta t}+\sigma^-a^\dagger{\rm e}^{{\rm i}\Delta t}), \qquad (23)$$

the operator a, $a^\dagger$ which annihilate and create a photon are then $a=(p-{\rm i}m\nu x)/\sqrt{2\hbar m\nu}$, $a^\dagger=(p+{\rm i}m\nu x)/\sqrt{2\hbar m\nu}$, where $[a,a^\dagger]=1$, $a|n\rangle=\sqrt{n}|n-1\rangle$ and $a^\dagger=\sqrt{n+1}|n+1\rangle$. Here $|n\rangle$ is an eigenstate of $a^\dagger a$. From $\sigma_z\sigma^\pm\sigma_z=-\sigma^\pm$ and $\mathcal{P}a\mathcal{P}=-a$ and $\mathcal{P}a^\dagger\mathcal{P}=-a^\dagger$, one can show that the equation (22) is pseudo-Hermitian with respect to $\mathcal{P}$ and σ_z [21]. So that, if the $H_{\rm tot}$ is pseudo-Hermitian with respect to $\mathcal{P}$ and σ_z, then $\sigma_z^{-1}\mathcal{P}=\sigma_z\mathcal{P}$ generates a symmetry of $H_{\rm tot}$, i.e. $[H_{\rm tot},\sigma_z\mathcal{P}]=0$ [18]. The inner product in the pseudo-Hermitian formalism is called the pseudo-inner product which is defined as

$$\langle\langle\psi_1\mid\psi_2\rangle\rangle_\eta:=\langle\psi_1\mid\eta\psi_2\rangle,$$

this definition is a possibly indefinite inner product on Hilbert space [20].

4. The eigenvalue problem

For the case $g_y=0$, the total Hamiltonian takes the JC form, and the JC Hamiltonian has already been solved completely. In this section, we consider the total Hamiltonian with the non-Hermitian part of the interaction only, i.e. $g_x=0$ and $t=0$

$$H_{\rm tot}=\frac{\hbar\omega}{2}\sigma_z+a^\dagger a\hbar\nu+{\rm i}\hbar g_y(\sigma^+a+\sigma^-a^\dagger). \qquad (24)$$

Let the two possible energy levels of the atom be denoted by $\pm\frac{\hbar\omega}{2}$, and the corresponding states by $|s\rangle$ $(s=e,g)$. Similarly, the number of quanta (photon) in the field oscillator will be n, and the corresponding state of the field by $|n\rangle$ $(n=0,1,2,\ldots)$. Obviously $\sigma_z|s\rangle=\lambda_s|s\rangle$, $(\lambda_e=1,\lambda_g=-1)$. We denote the state of the total Hamiltonian by $|s,n\rangle$ and it is well known that the projection operators $\sigma^\pm$ have the following usual properties when they act on the states $|s,n\rangle$, $\sigma^+|e,n\rangle=0$, $\sigma^-|g,n\rangle=0$, $\sigma^+|g,n\rangle=|e,n\rangle$, $\sigma^-|e,n\rangle=|g,n\rangle$. Although, $H_{\rm tot}|g,0\rangle=-\frac{\hbar\omega}{2}|g,0\rangle$, it is easily seen that $|g,0\rangle$ is a ground state of the Hamiltonian.

By applying $H_{\rm tot}$ on $|e,n\rangle$, and then $|g,n+1\rangle$ we have

$$H_{\rm tot}|e,n\rangle=\hbar\left(n\nu+\frac{\omega}{2}\right)|e,n\rangle+\mathrm{i}\hbar g_y\sqrt{n+1}|g,n+1\rangle,$$
$$H_{\rm tot}|g,n+1\rangle=\hbar\left((n+1)\nu-\frac{\omega}{2}\right)|g,n+1\rangle+\mathrm{i}\hbar g_y\sqrt{n+1}|e,n\rangle. \quad (25)$$

So that the Hamiltonian matrix in the $|s,n\rangle$ basis is given by

$$H_{\rm tot}=\begin{bmatrix}\hbar(n\nu+\frac{\omega}{2}) & \mathrm{i}\hbar g_y\sqrt{n+1}\\ \mathrm{i}\hbar g_y\sqrt{n+1} & \hbar((n+1)\nu-\frac{\omega}{2})\end{bmatrix}. \quad (26)$$

The eigenvalues of this Hamiltonian matrix are given by

$$E_n^{(1,2)}=\tfrac{1}{2}\left[(2n+1)\hbar\nu\pm\hbar\sqrt{(\nu-\omega)^2-4g_y^2(n+1)}\right]. \quad (27)$$

Note, these eigenvalue are real provided

$$(\nu-\omega)^2\geqslant 4g_y^2(n+1). \quad (28)$$

So, by defining $4g_y^2(n+1)=(\nu-\omega)^2\sin^2(\theta_n)$, we obtain

$$E_n^{(1)}=\frac{\hbar}{2}\left((2n+1)\nu+(\nu-\omega)\cos(\theta_n)\right),$$
$$E_n^{(2)}=\frac{\hbar}{2}\left((2n+1)\nu-(\nu-\omega)\cos(\theta_n)\right), \quad (29)$$

and then the two eigenstates corresponding to two eigenvalues are

$$|\psi_n^{(1)}\rangle=|A_n|\left[\sin\left(\frac{\theta_n}{2}\right)|e,n\rangle-\mathrm{i}\cos\left(\frac{\theta_n}{2}\right)|g,n+1\rangle\right],$$
$$|\psi_n^{(2)}\rangle=|B_n|\left[\cos\left(\frac{\theta_n}{2}\right)|e,n\rangle-\mathrm{i}\sin\left(\frac{\theta_n}{2}\right)|g,n+1\rangle\right], \quad (30)$$

where A_n and B_n are normalization constants,

$$|A_n|^2=-|B_n|^2=\frac{1}{\sin^2(\theta_n/2)-\cos^2(\theta_n/2)}. \quad (31)$$

It is easily seen that the eigenstates in (30) satisfy the pseudo-inner product with respect to σ_z, as

$$\langle\langle\psi_n^{(i)}\mid\psi_m^{(j)}\rangle\rangle_{\sigma_z}=\langle\psi_n^{(i)}\mid\sigma_z\psi_m^{(j)}\rangle=\langle\psi_n^{(i)}\mid\phi_m^{(j)}\rangle=\delta_{nm}\delta^{ij}. \quad (32)$$

Here $|\phi_m^{(i)}\rangle=\sigma_z|\psi_m^{(i)}\rangle$ and $\langle\cdot|\cdot\rangle$ is the original inner product. One can find that $|\phi_n^{(i)}\rangle$s are the eigenkets of $H^\dagger$ with eigenvalue $E_n^{(i)}$. Therefore the Hamiltonian, $H_{\rm tot}$, is called the σ_z-pseudo-Hermitian Hamiltonian and has a complete set of biorthonormal eigenkets $\{|\psi_n^{(i)}\rangle,|\phi_n^{(i)}\rangle\}$ in which

$$H_{\rm tot}|\psi_n^{(i)}\rangle=E_n^{(i)}|\psi_n^{(i)}\rangle,$$
$$H_{\rm tot}^\dagger|\phi_n^{(i)}\rangle=E_n^{(i)}|\phi_n^{(i)}\rangle,$$
$$\sum_n\sum_i|\psi_n^{(i)}\rangle\langle\phi_n^{(i)}|=\sum_n\sum_i|\phi_n^{(i)}\rangle\langle\psi_n^{(i)}|=I. \quad (33)$$

The pseudo-inner-product, (32), defining a new Hilbert space $\mathcal{H}_\eta$, then the Hamiltonian $H_{\rm tot}$ (which is the non-Hermitian with respect to the original scalar product $\langle\cdot|\cdot\rangle$) is Hermitian with respect to the new one

$$\langle\langle\psi_1\mid H\psi_2\rangle\rangle_{\sigma_z}=\langle\psi_1\mid\sigma_z H\psi_2\rangle=\langle\psi_1\mid H^\dagger\sigma_z\psi_2\rangle=\langle H\psi_1\mid\sigma_z\psi_2\rangle=\langle\langle H\psi_1\mid\psi_2\rangle\rangle_{\sigma_z}. \quad (34)$$

It is known that these eigenstates are eigenstates of the operator $\mathcal{P}\sigma_z$ also

$$\mathcal{P}\sigma_z|\psi_n^{(1,2)}\rangle=(-1)^n|\psi_n^{(1,2)}\rangle, \quad (35)$$

because $\mathcal{P}|s,n\rangle=(-1)^n|s,n\rangle$ and $\sigma_z|s,n\rangle=\lambda_s|s,n\rangle$ ($\lambda_e=1$, $\lambda_g=-1$). These properties are due to, $\mathcal{P}\sigma_z$ invariant Hamiltonian, i.e. $[\mathcal{P}\sigma_z,H_{\rm tot}]=0$. Furthermore, we can easily show that $[H,\mathcal{T}]\neq 0$, where $\mathcal{T}$ is the time reversal operator for spin-$\frac{1}{2}$ system which is given by $\mathcal{T}=-\mathrm{i}\sigma_y K$, where K is the complex conjugate operator and $\tilde{\sigma}$ is the Pauli matrix.

By solving equations (30), one can obtain the $|e,n\rangle$ and $|g,n+1\rangle$ states as:

$$|e,n\rangle=\left[|A_n|\sin\left(\frac{\theta_n}{2}\right)|\psi_n^{(1)}\rangle+|B_n|\cos\left(\frac{\theta_n}{2}\right)|\psi_n^{(2)}\rangle\right],$$
$$|g,n+1\rangle=\mathrm{i}\left[|A_n|\sin\left(\frac{\theta_n}{2}\right)|\psi_n^{(2)}\rangle+|B_n|\cos\left(\frac{\theta_n}{2}\right)|\psi_n^{(1)}\rangle\right]. \quad (36)$$

It is seen that for the case when the system has one photon, the total system (field + two-level atom) has two states such as

$$|e,0\rangle=\left[|A_0|\sin\left(\frac{\theta_0}{2}\right)|\psi_0^{(1)}\rangle+|B_0|\cos\left(\frac{\theta_0}{2}\right)|\psi_0^{(2)}\rangle\right],$$
$$|g,1\rangle=\mathrm{i}\left[|A_0|\sin\left(\frac{\theta_0}{2}\right)|\psi_0^{(2)}\rangle+|B_0|\cos\left(\frac{\theta_0}{2}\right)|\psi_0^{(1)}\rangle\right]. \quad (37)$$

where e and g denote the excitation and ground states of the free atom, respectively.

5. Evolution of the model

In this section, we shall solve the evolution of the atom–field system described by Hamiltonian (22). We first proceed to solve the equation of motion for $|\psi\rangle$, i.e.

$$\mathrm{i}\hbar|\dot{\psi}\rangle=\mathcal{V}|\psi\rangle. \quad (38)$$

At any time t, the state vector $|\psi(t)\rangle$ is a linear combination of states $|e,n\rangle$ and $|g,n+1\rangle$. Here equation (38) is the Schrödinger equation in the interaction picture in which the interaction is a pseudo-interaction as

$$\mathcal{V}=\sigma_z H_{\rm int}. \quad (39)$$

We can express the vector as

$$|\psi(t)\rangle=\sum_n(c_{e,n}(t)|e,n\rangle+c_{g,n+1}|g,n+1\rangle). \quad (40)$$

The pseudo-interaction (39) can only cause transitions between the states $|e,n\rangle$ and $|g,n+1\rangle$. By substituting (40) and (39) in (38) and projecting the resulting equation on to $\langle e, n|$ and $\langle g, n+1|$ respectively, we obtain

$$\begin{cases} \dot{c}_{e,n}(t) = g_y\sqrt{n+1}c_{g,n+1}(t)\,\mathrm{e}^{\mathrm{i}\Delta t}, \\ \dot{c}_{g,n+1}(t) = -g_y\sqrt{n+1}c_{e,n}(t)\,\mathrm{e}^{-\mathrm{i}\Delta t}. \end{cases} \quad (41)$$

A general solution for the probability amplitude is

$$\begin{cases} c_{e,n}(t) = \left\{c_{e,n}(0)\left[\cos\frac{\Omega_n t}{2} - \frac{\mathrm{i}\Delta}{\Omega_n}\sin\frac{\Omega_n t}{2}\right] + \frac{2g_y\sqrt{n+1}}{\Omega_n}c_{g,n+1}(0)\sin\frac{\Omega_n t}{2}\right\}\mathrm{e}^{\mathrm{i}(\Omega_n t/2)}, \\ c_{g,n+1}(t) = \left\{c_{g,n+1}(0)\left[\cos\frac{\Omega_n t}{2} + \frac{\mathrm{i}\Delta}{\Omega_n}\sin\frac{\Omega_n t}{2}\right] - \frac{2g_y\sqrt{n+1}}{\Omega_n}c_{e,n}(0)\sin\frac{\Omega_n t}{2}\right\}\mathrm{e}^{-\mathrm{i}(\Omega_n t/2)}, \end{cases} \quad (42)$$

where

$$\Omega_n = \sqrt{\Delta^2 + 4g_y^2(n+1)}, \quad (43)$$

so that for an initial condition such as $c_{e,n} = c_n(0)$ and $c_{g,n+1} = 0$, we have

$$\begin{cases} c_{e,n}(t) = c_n(0)\left[\cos\frac{\Omega_n t}{2} - \frac{\mathrm{i}\Delta}{\Omega_n}\sin\frac{\Omega_n t}{2}\right]\mathrm{e}^{\mathrm{i}(\Omega_n t/2)}, \\ c_{g,n+1}(t) = -c_n(0)\frac{2g_y\sqrt{n+1}}{\Omega_n}\sin\frac{\Omega_n t}{2}\mathrm{e}^{-\mathrm{i}(\Omega_n t/2)}. \end{cases} \quad (44)$$

The probability $P(n)$ that there are n photons in the field at time t is therefore obtained by the trace over the atomic state, i.e.

$$\begin{aligned} P(n) &= |c_{e,n}|^2 + |c_{g,n}|^2 \\ &= |c_n(0)|^2\left[\cos^2\frac{\Omega_n t}{2} + \frac{\Delta^2}{\Omega_n^2}\sin^2\frac{\Omega_n t}{2}\right] \\ &\quad + |c_{n-1}(0)|^2\left(\frac{4g_y^2 n}{\Omega_{n-1}^2}\right)\sin^2\frac{\Omega_{n-1}t}{2}. \end{aligned} \quad (45)$$

We plot $P(n)$ for an initial coherent state $\rho_{nn}(0) = \frac{\bar{n}^n e^{-\bar{n}}}{n!}$. Another important quantity is the inversion $W(t)$ which is defined as

$$W(t) = \sum_n [|c_{e,n}|^2 - |c_{g,n+1}|^2]. \quad (46)$$

By substituting (44) in (46), we obtain

$$W(t) = \sum_n |c_n|^2\left[\frac{\Delta^2}{\Omega_n^2} + \frac{4g_y^2(n+1)}{\Omega_n^2}\cos\Omega_n t\right]. \quad (47)$$

For the initial vacuum field $|c_n(0)|^2 = \delta_{n,0}$, which is equivalent to the monochromic field, we have

$$W(t) = \frac{1}{\Omega^2}[\Delta^2 + 4g_y^2\cos\Omega t], \quad (48)$$

which explains the Rabi oscillations. See figures 1(a) and (b).

It is seen that this sinusoidal Rabi oscillation collapses and then revives. The time t_{R}, t_{c} and t_{r} associated with Rabi oscillations, the collapse of these oscillations and their revival,

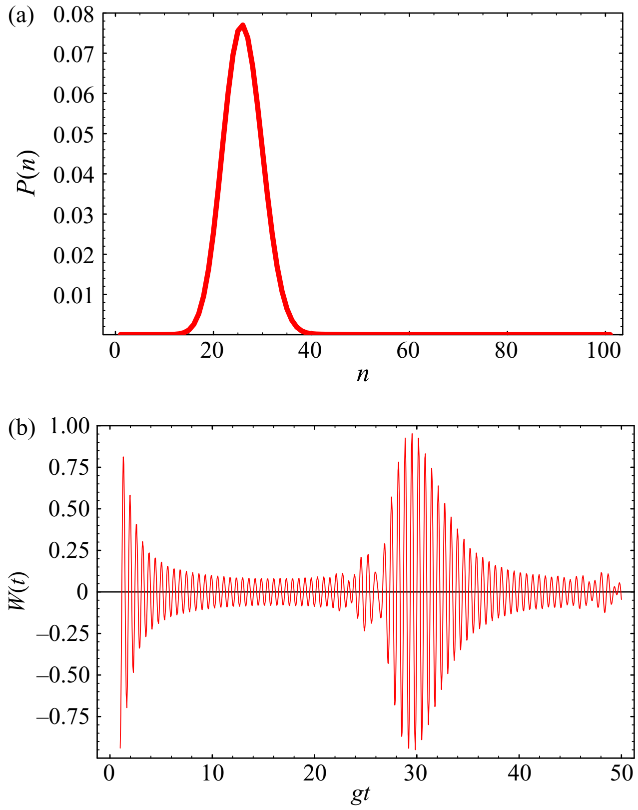

Figure 1. (a) $P(n)$ as function of n. (b) $W(t)$ as a function of gt. We consider that $\bar{n} = 25$, $\Delta = 0$ and the distribution of photons is $\rho_{\mathrm{nn}}(0) = \frac{\bar{n}^n e^{-\bar{n}}}{n!}$. These results and Zubari's results [1] are in good agreement.

respectively, can be determined from (46) in the limit $\bar{n} \gg 1$. The t_{R} is given as $t_{\mathrm{R}} \sim \frac{1}{\Omega_n} = \frac{1}{\sqrt{\Delta^2 + 4g_y^2\bar{n}}}$.

Another choice for pseudo interaction is

$$\mathcal{V} = \mathcal{P}H_{\mathrm{int}}, \quad (\mathcal{P}\text{ is parity}), \quad (49)$$

so one can solve equation (38) and arrive at

$$\begin{cases} \dot{c}_{e,n}(t) = (-1)^n g_y\sqrt{n+1}c_{g,n+1}(t)\,\mathrm{e}^{\mathrm{i}\Delta t}, \\ \dot{c}_{g,n+1}(t) = (-1)^{n+1}g_y\sqrt{n+1}c_{e,n}(t)\,\mathrm{e}^{-\mathrm{i}\Delta t}, \end{cases} \quad (50)$$

where we use $\mathcal{P}|s, m\rangle = (-1)^m|s, m\rangle$, $(s = e, g)$. It is clearly seen that for an initial condition such as $c_{e,n}(0) = c_n(0)$ and $c_{g,n+1}(0) = 0$, we have

$$\begin{cases} c_{e,n}(t) = c_n(0)\left[\cos\frac{\Omega_n t}{2} - \frac{\mathrm{i}\Delta}{\Omega_n}\sin\frac{\Omega_n t}{2}\right]\mathrm{e}^{\mathrm{i}(\Delta t/2)}, \\ c_{g,n+1}(t) = -c_n(0)\frac{2(-1)^n g_y\sqrt{n+1}}{\Omega_n}\sin\frac{\Omega_n t}{2}\mathrm{e}^{\mathrm{i}(\Delta t/2)}, \end{cases} \quad (51)$$

where Ω_n has been defined in equation (43). It is clearly seen that for this case the probability $P(n)$ is obtained in equation (45) and one can arrive at the other result which is obtained for $\sigma_z H_{\mathrm{int}}$.

6. Conclusion

In ordinary quantum mechanics, the Hamiltonian of a physical system must be Hermitian. In this paper, we have introduced a new model of investigation of the interaction of radiation and matter in which there is no need to have a Hermitian Hamiltonian. In fact, unlike the ordinary quantum mechanics, we have not constrained ourselves to adapt $\hat{\epsilon}$ and $\hat{\epsilon}^{\star}$ simultaneously. We have not satisfied the hermiticity condition and we have used only ϵ in the interaction term which is non-Hermitian Hamiltonian. Consequently, we find a non-Hermitian interaction between radiation and matter. Indeed this interaction is a physical phenomenon, because of the following:

- Considering the time evolution of the model in the interaction picture, we have obtained the probability and population inverse of the two-level atom in EM field. Our results are in excellent agreement with the one which was obtained from ordinary quantum mechanics or experimental data.
- The total Hamiltonian has real energy eigenvalues.

Therefore, we find the non-Hermitian Hamiltonian which one can use for obtaining the physical results for interaction between light and matter.

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